

## Application of Reimann Integrals

1. **Area of a plane figure** (in Cartesian coordinates):

a.  $D = \{(x;y): a \leq x \leq b \quad 0 \leq y \leq f(x)\}$   $|D| = \int_a^b f(x) dx$

b.  $D = \{(x;y): a \leq x \leq b \quad f(x) \leq y \leq 0\}$   $|D| = - \int_a^b f(x) dx$

c.  $D_X = \{(x;y): a \leq x \leq b \quad f(x) \leq y \leq g(x)\}$   $|D_X| = \int_a^b (g(x) - f(x)) dx$

d.  $D_Y = \{(x;y): c \leq y \leq d \quad h(y) \leq x \leq k(y)\}$   $|D_Y| = \int_c^d (k(y) - h(y)) dy$

Example: Calculate the area of the figure between the curve  $y = -x^2 + 2x - 2$  and the straight lines  $y = x + 1$   $x = -1$   $x = 2$

2. **Length of curve:**

$y = f(x)$  – continues  $l = \int_a^b \sqrt{1 + f'(x)^2} dx$

Example: Calculate length of the chain line  $y = \frac{a}{2} \left( e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)$  i.e.  $y = ach \frac{x}{a}$ , where

$0 < x < b$ .

3. **The lateral surface area and the volume of a solid of revolution:**

$y = f(x)$  – continues,  $f(x) \geq 0$

**Volume:**  $|V| = \pi \int_a^b f^2(x) dx$

Example: Calculate volume of torus powstały przez obrót koła: torus formed by the rotation of circle:  
 $x^2 + (y-h)^2 \leq r^2$

**Lateral surface area:**  $|S| = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} dx$

Example: Calculate the Lateran surface area of this torus.