Application of Riemann Integrals

1. **Area of a plane figure** (in Cartesian coordinates):

   a. D={(x;y): a≤x≤b 0≤y≤f(x)} $|D| = \int_a^b f(x)dx$

   b. D={(x;y): a≤x≤b f(x)≤y≤0} $|D| = -\int_a^b f(x)dx$

   c. D_X={(x;y): a≤x≤b f(x)≤y≤g(x)} $|D_X| = \int_a^b (g(x) - f(x))dx$

   d. D_Y={(x;y): c≤y≤d h(y)≤x≤k(y)} $|D_Y| = \int_c^d (k(y) - h(y))dy$

Example: Calculate the area of the figure limited by the curve $y=-x^2+2x-2$ and the straight lines y=x+1 x=-1 x=2

2. Length of a curve (an arc length):

y=f(x) – continuous function $l = \int_a^b \sqrt{1 + f'^2(x)}dx$

Example: Calculate the length of the chain line $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$ i.e. $y = ach\frac{x}{a}$, where 0<x<b.

3. Lateral surface area and volume of a solid of revolution:

y=f(x) – continuous function, f(x)≥0

Volume: $|V| = \pi\int_a^b f^2(x)dx$

Example: Calculate the volume of the torus formed by the rotation of the circle: $x^2+(y-h)^2\le r^2$

Lateral surface area: $|S| = 2\pi\int_a^b f(x)\sqrt{1 + f'^2(x)}dx$

Example: Calculate the lateral surface area of this torus.