LIST 1. Definite Integrals

<u>Task 1</u>: Calculate definite integrals by parts:

a. $\int_{2}^{3}(x-1)^3 \ln^2(x-1)dx$; b. $\int_{0}^{\frac{\pi}{2}}(3-7x^2)\cos 2xdx$; c. $\int_{0}^{1}x^2 e^{3x}dx$; d. $\int_{\frac{\sqrt{3}}{3}}^{1}\frac{\arctan x}{x^2}dx$.

<u>Task 2.</u> Using integration by substitution calculate definite integrals:

a. $\int_{1}^{2}\frac{dx}{x\sqrt{x^2+1}}$; b. $\int_{0}^{\frac{1}{2}}\frac{(\arccos x)^2-1}{\sqrt{1-x^2}}dx$; c. $\int_{2}^{3}\frac{1+\ln x}{x}dx$ d. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1-\cos x}{(x-\sin x)^2}dx$.

<u>Task 3</u>: Calculate the areas of the surfaces limited by the curves:

a. $y=(x-2)^2$, $y=4x-8$; b. $y=4-x^2$, $y=x^2-2x$; c. $y=-x^2+2x+3$, $y=x^2-4x+3$;

d. $y=x\sqrt{9-x^2}$, $y=0$; e. $y=(x+1)^2$, $y^2=x+1$; f. $y=\frac{1}{x}$, $y=-x+4$; g. $y=\frac{27}{x^2+9}$, $y=\frac{x^2}{6}$;

h. $y=e^x$, $y=e^{-x}$, $x=1$; i. $y=e^x$, $y=e^{2x}-2$, $x=0$; j. $y=\ln(x+1)$, $y=\ln x$, $x=2$, $y=0$.

<u>Task 4</u>: Calculate the length of the given curves:

a. $y=x^2$, $0\le x\le 1$; b. $y=\frac{1}{2}(e^x+e^{-x})$, $0\le x\le 1$; c. $y=2x$, $x\in[-1,0]$;

d. $y=\ln x$, $\sqrt{3}\le x\le\sqrt{15}$; e. $y=2\ln(4-x^2)$, $0\le x\le 1$; f. $y=\sqrt{1-x^2}+\arcsin x$, $0\le x\le\frac{7}{9}$.

<u>Task 5</u>: Calculate the volume of the solids of revolution (rotation around the x-axis):

a. $\frac{4}{x}\le y\le 5-x$, $1\le x\le 4$; b. $x^2\le y\le\sqrt{x}$, $0\le x\le 1$; c. $0\le y\le\sqrt{x}e^{-x}$, $0\le x\le 4$.

<u>Task 6</u>: Calculate the lateral area of the solids of revolution (rotation around the x-axis):

a. $y=\frac{1}{3}x^3$, $0\le x\le 1$; b. $y^2=8x$, $0\le x\le 6$; c. $y=\sqrt{4-x^2}$, $-\frac{1}{2}\le x\le\frac{3}{2}$.