LIST 1. Fundamentals of Logic.

1. Do you think that the following quote is the logical statement? If yes, give its logical value.

a) Is the number π^2 greater than 10?
b) The number π^2 is greater than 10.
c) The number π^2 is less than 10.
d) The natural number n is less than 10.
e) There is at least one natural number less than 10.
f) Each natural number is less than 10.
g) Madrid is the capital of Ukraine.
h) The number of $10^{100} + 1$ is divisible by 2;
i) $x^2=4$.

2. Specify a logical value of the complex sentences given below:
   a) $(2^4=16)\wedge\left(\sqrt{99}=9\right)$.
   b) $\sim(4\cdot 6=23)$.
   c) $(7>6)\vee(3<2)$
   d) $(|-3|=-3)\vee(\cos\pi=-1)$,
   e) $(\log_2 8=3)\Rightarrow(\sin\pi=2)$,
   f) $(e=3)\Rightarrow(\pi=1)$,
   g) It is not true that the function $f(x)=x^2$ is increasing on $\boldsymbol{R}$.
   h) $(-1)^{44}=-1$ or 2011 is an odd number.

3. Using the "0-1" method check if the following sentences are the laws of propositional calculus (tautologies):

(a) $p\Leftrightarrow p$, (b) $(p\vee q)\Rightarrow q$, (c) $(p\wedge q\Rightarrow r)\Leftrightarrow[p\Rightarrow(q\Rightarrow r)]$,

(d) $p\Rightarrow(\sim p\vee q)$, (e) $\sim p\Leftrightarrow\sim(p\wedge\sim q)$, (f) $(p\Rightarrow q)\Rightarrow(q\Rightarrow p)$,

(g) $\sim(p\Rightarrow\sim p)$, (h) $\sim(p\Rightarrow q)\Rightarrow(p\Rightarrow\sim q)$, (i) $(p\vee q)\Rightarrow[(p\Rightarrow q)\Rightarrow q]$

4. a) Define the conjunction using the alternative and the negation.
   b) Define the alternative using the conjunction and the negation.
   c) Define the alternative using the implication and the negation.

5. Which sentence is true, which is false? Write the negation of each sentence.

(a) $\forall x\in R, (x^2-2\geq 0)$ (b) $\exists x\in R, (x^2-2\geq 0)$ (c) $\forall x\in R\ \forall y\in R, (x+y=0)$
(d) $\forall x\in R\ \exists y\in R, (x+y=0)$ (e) $\exists x\in R\ \exists y\in R, (x+y=0)$
(f) $\exists x\in R\ \forall y\in R, (y<x^2+1)$ (g) $\forall y\in R\ \exists x\in R, (y<x^2+1)$