LIST 1. The Double Integrals.

1. Find the double integral: $\iint_D x\sin(xy)dxdy$, *where* $D=[0,1]\times[\pi,2\pi]$ is the rectangle.

2. The double integral change to the sum of multiplications of Riemann integrals:

$$\iint_D xy\ln\frac{x}{y}dxdy; \quad where \quad D=[1,e]\times[1,2].$$

3. The double integral $\iint_D f(x,y)dxdy$ change to the iterated integral, where D is the region limited by the lines:

a) $x^2+y^2=4$, $y=2x-x^2$, $x=0$ $(x,y\geq 0)$; b) $x^2-4x+y^2+6y-51=0$.

4. Calculate integrals:

a) $\iint_D \min(x,y)dxdy;$ *where* $D=[0,1]\times[0,2]$;

b) $\iint_D |x-y|dxdy;$ *where* $D=\{(x,y)\in R^2 : x\geq 0,\ 0\leq y\leq 3-2x\}$.

5. In the integrals below change the order of integration:

$$\text{a) } \int_0^4 dx \int_{\sqrt{4x-x^2}}^{2\sqrt{x}} f(x,y)dy; \qquad \text{b) } \int_{-\sqrt{2}}^{\sqrt{2}} dy \int_{y^2-1}^{\frac{y^2}{2}} f(x,y)dx.$$

6. Calculate mean value of the function $f(x,y)=x+y$ over the area D: $0\leq y\leq \pi,\ 0\leq x\leq \sin y$.

7. Calculate integrals:

a) $\iint_D xydxdy$; *where* D: $x\geq 0\ \ 1\leq x^2+y^2\leq 2$;

b) $\iint_D (x^2+y^2)dxdy$; *where* D: $y\geq 0\ \ y\leq x^2+y^2\leq x$.

8. Using double integration find the surface area of the region limited by the curves:

a) $y^2=4x$, $x+y=3$; b) $x^2+y^2-2y=0$, $x^2+y^2-4y=0$.

9. Using double integrals calculate the volume of the solids limited by the surfaces:

a) $x^2+y^2-2y=0$, $z=x^2+y^2$, $z=0$; b) $x^2+y^2+z^2-2z=0$.

10. Using double integration calculate the surface area of the surfaces:

a) $z=x^2+y^2$, $x^2+y^2\leq 1$; b) $x^2+y^2+z^2=R^2$, $x^2+y^2-Rx\leq 0$, $z\geq 0$; c) $z=\sqrt{x^2+y^2}$, $1\leq z\leq 2$.

11. Calculate the mass of the area $D=\{(x,y)\in R^2 : 1\leq x^2+y^2\leq 4,\ y\geq 0\}$. Surface density $\sigma(x,y)=|x|$.

12. Find the coordinates of the center of mass of homogenous area (surface density $\sigma(x,y)=1$): $D=\{(x,y)\in R^2 : 0\leq x\leq \pi,\ 0\leq y\leq \sin^2 x\}$

13. Calculate the moment of inertia with respect to x-axis of the area:

$$D=\{(x,y)\in R^2 : x^2+y^2\leq R^2,\ y\geq 0\}$$

Surface density $\sigma(x,y)=\sqrt{x^2+y^2}$.