

LIST 1. The de L'Hospital's Rule and the Taylor's Theorem.

Task 1: Using the de L'Hospital's rule calculate the limits:

$$\begin{aligned}
 & a) \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\ln(\cos 2x + 1)}{\ln \cos x}; \quad b) \lim_{x \rightarrow 0^+} \frac{\frac{\pi}{x}}{\cot \frac{\pi x}{2}}; \quad c) \lim_{x \rightarrow \infty} \frac{\ln(2^x + 1)}{x}; \quad d) \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt[3]{x}}; \\
 & e) \lim_{x \rightarrow 1} \frac{\ln \sin \frac{\pi x}{2}}{\ln x}; \quad f) \lim_{x \rightarrow 0^+} \frac{chx - 1}{1 - \cos x}; \quad g) \lim_{x \rightarrow 0} \frac{x - \arctan x}{x^2}; \quad h) \lim_{x \rightarrow 1} \frac{x^{10} - 10x + 9}{x^5 - 5x + 4}; \\
 & i) \lim_{x \rightarrow \infty} x \operatorname{arccot} x; \quad j) \lim_{x \rightarrow 0^+} x \ln x; \quad k) \lim_{x \rightarrow \pi^-} (\pi - x) \tan \frac{x}{2}; \quad l) \lim_{x \rightarrow 0^-} (1 - \cos x) \cot x; \quad m) \lim_{x \rightarrow 0^+} \left(\frac{1}{1 - \cos x} - \frac{1}{x^2} \right); \\
 & n) \lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \cot x \right); \quad o) \lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right); \quad p) \lim_{x \rightarrow 0^+} x^{\frac{3}{4 + \ln x}}; \quad q) \lim_{x \rightarrow 0^+} (-\ln x)^x; \quad r) \lim_{x \rightarrow \frac{\pi}{2}^-} (\tan x)^{\cos x}; \\
 & s) \lim_{x \rightarrow 0^+} (\cot x)^{\frac{1}{\ln x}}; \quad t) \lim_{x \rightarrow 0^+} (1 + x^2)^{1/x}; \quad u) \lim_{x \rightarrow 0^+} (1 + x)^{\ln x}; \quad w) \lim_{x \rightarrow \infty} \left(\frac{2}{\pi} \arctan x \right)^x.
 \end{aligned}$$

Task 2: Write the 4th order Taylor's formula for the functions $f(x)$ at the points x_0 :

$$\begin{aligned}
 a. \quad & f(x) = e^x, \quad x_0 = -1; & b. \quad & f(x) = \frac{x}{x-1}, \quad x_0 = 2.
 \end{aligned}$$

<https://www.desmos.com/calculator/2ztwe1s1oj>

<https://www.desmos.com/calculator/zhib5ihyx>

Task 3: Write the n^{th} order Taylor's formula for the functions $f(x)$ at the points x_0 :

$$\begin{aligned}
 a. \quad & f(x) = \frac{1}{x}, \quad x_0 = -1; & b. \quad & f(x) = \sqrt{x}, \quad x_0 = 4.
 \end{aligned}$$

Task 4: Estimate the absolute errors of the approximate formulas:

$$\begin{aligned}
 a) \quad & e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}, \quad 0 \leq x \leq 1; & b) \quad & \tan x \approx x + \frac{x^3}{3}, \quad |x| \leq 0,1.
 \end{aligned}$$

Task 5: Find approximate values of the numbers below with the absolute error smaller than d .

$$\begin{aligned}
 a) \quad & \frac{1}{\sqrt[4]{e}}, \quad d = 0,01; & b) \quad & \cos(10^\circ), \quad d = 0,001.
 \end{aligned}$$