LIST 10. Analytic Geometry

1. Calculate the length and the dot product of vectors: **u**=(1,-2,5) and **w**=(3,-1,0).

2. Using dot product calculate angles between:
   a) vectors: **u**=(-3,0,4) and **w**=(0,1,-2);
   b) bisector of X-axis and Y-axis and bisector of Y-axis and Z-axis;
   c) diagonals of the parallelepiped spanned by the vectors: **u**=(1,2,3), **v**=(-1,0,2), **w**=(3,1,5).

3. Calculate the length of the orthogonal projection of vector $\vec{a}=\left(\sqrt{2},\sqrt{3},-\sqrt{5}\right)$ on vector $\vec{b}=\left(-\sqrt{8},0,\sqrt{5}\right)$.

4. Calculate the cross product of vectors: **u**=(-3,2,0) and **w**=(1,5,-2).

5. Calculate the surface area of:
   a) the parallelogram spanned by vectors **u**=(-3,2,0) and **w**=(1,5,-2);
   b) the triangle with vertices A=(1,-1,3), B=(0,2,-3), C=(2,2,1).

6. Calculate the volume of the given polyhedra**:**
   a) the parallelepiped spanned by the vectors: **u**=(0,0,1), **w**=(-1,2,3), **v**=(2,5,-1);
   b) the tetrahedron with vertices A(1,1,1), B(1,2,3), C(2,3,-1), D(-1,3,5).

7. Check whether:
   a) the vectors **u**=(1,-1,1), **w**=(-1,3,-5), **v**=(4,-2,0) are coplanar;
   b) the points A(0,0,0), B(-1,2,3), C(2,3,-4), D(2,-1,5) are coplanar.

8. Write a general equation of the planes satisfying the following conditions:
   a) the plane containing the points A(0,0,0), B(1,2,3) and C(-1,-3,5);
   b) the plane containing the points A(1,-3,4) and B(2,0,-1) and perpendicular to the plane: 2x+y-z+3=0;
   c) the plane containing the point A(1,-1,3) and parallel to the vectors **u**=(1,1,3), **w**=(0,1,-1);
   d) the plane containing the point A(0,3,0) and parallel to the plane π: 3x+2y+2z-1=0;
   e) the plane containing the point A(2,1,-3) and perpendicular to the planes π_1: x+y+z=0 and π_2: 2x-y-z=0.

9. Write parametric and directional equations of straight lines satisfying the following conditions:
   a) the straight line going through two points A(1,0,6) and B(-2,2,4);
   b) the straight line contains the point A(0,-2,3) and is perpendicular to the plane π: 3x-y+2z-6=0;
   c) the straight line contains the point A(1,1,-1) and is perpendicular to the vectors: **u**=(-2,1,-1), **w**=(1,0,1);

10. Calculate the distance between:
   a) two planes π_1: x-2y+2z+5=0 and π_2: 3x-6y+6z-3=0;

b) two skew lines l_1: $\frac{x-1}{-1}=\frac{y-3}{1}=\frac{z}{-6}$, l_2: $\frac{x-4}{2}=\frac{y}{-1}=\frac{z-1}{2}$;

c) the straight line l: $\begin{cases} x=2+t \\ y=-3+2t \\ z=2-t \end{cases}$, t∈R, and the plane π: 2x+y+4z=0.

11. Calculate an angle between:

a) the straight line l: $\frac{x-3}{2}=\frac{y-1}{0}=\frac{z+2}{-3}$ and the plane π: x-z=0;

b) two planes π_1: x-2y+3z-5=0 and π_2: 2x+y-z+3=0;

c) two straight lines l_1: $\begin{cases} x=1-t \\ y=-2+t \\ z=3t \end{cases}$, t∈R, and l_2: $\begin{cases} x=3-2t \\ y=4-t \\ z=1+3t \end{cases}$, t∈R.

12. Find the point symmetric to the point P (2,3, -1) with respect to:

a) the point S(1,-1,2);

b) the straight line l: $\begin{cases} x+y=0 \\ y+z=0 \end{cases}$;

c) the plane π: 2x-y+z-6=0.

13. Name and sketch the given surface using the method of determining of the level curves and the method of the traces of the surface:

a) $4(x^2+y^2)+z^2=1$; b) $z=1-4(x^2+y^2)$; c) $z=-4x^2+y^2$; d) $y=-4x^2$;

e) $x^2+y^2-z^2=-1$; f) $x^2+y^2-z^2=1$; g) $x^2+y^2-z^2=0$.