

LIST 2. Multivariable Functions.

Task 1. Determine and draw domains of two variables functions:

$$a. f(x, y) = \sqrt{x \sin y}; \quad b. f(x, y) = \sqrt{x-1} + \ln(2y+3); \quad c. f(x, y) = \ln(xy);$$

$$d. f(x, y) = \sqrt{x-y} + \sqrt{x+y}; \quad e. f(x, y) = \arcsin \frac{y-1}{x}; \quad f. f(x, y) = \ln \frac{x^2 + y^2 - 4}{9 - x^2 - y^2}.$$

Task 2: Calculate the first-order partial derivatives of the functions given below. Write their gradients.

$$a. f(x, y) = \frac{x}{y} + \frac{y}{x}; \quad b. f(x, y) = \ln(x^2 + xy + y^2); \quad c. f(x, y) = \sqrt{x} \sin \frac{y}{x}; \quad d. f(x, y) = x\sqrt{y} + \frac{y}{\sqrt[3]{x}};$$

$$e. f(x, y, z) = \arctan \frac{y}{x} + xz; \quad f. f(x, y, z) = x^{\frac{y}{z}}; \quad g. f(x, y, z) = x^2 y - \sqrt{x^2 + 5z^4}; \quad h. f(x, y, z) = \frac{y}{\sqrt{x^2 + y^2 + z^2}}.$$

Task 3. Write the equation of the tangent plane and the normal line to the graph of the function $f(x, y)$ in the point (x_0, y_0, z_0) :

$$a) f(x, y) = x^2 \sqrt{y+1}, \quad (x_0, y_0, z_0) = (1, 3, z_0); \quad b) f(x, y) = \sqrt{9 - x^2 - y^2}, \quad (x_0, y_0, z_0) = (\sqrt{2}, -\sqrt{3}, z_0);$$

$$c) f(x, y) = x^y, \quad (x_0, y_0, z_0) = (2, 4, z_0); \quad d) f(x, y) = e^{x+2y}, \quad (x_0, y_0, z_0) = (2, -1, z_0).$$

Task 4: Calculate the gradient and the directional derivative at the given point $M(x_0, y_0, z_0)$ and in the specified direction of the given vectors \mathbf{u} :

$$a. f(x, y, z) = (x^2 + y^2 + z^2)^{3/2}, \quad (x_0, y_0, z_0) = (1, 1, 1), \quad \mathbf{u} = \left(\frac{\sqrt{3}}{3}, \frac{-\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right);$$

$$b. f(x, y, z) = x(\ln y - \arctan z), \quad (x_0, y_0, z_0) = (-2, 1, 1), \quad \mathbf{u} = \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right);$$

$$c. f(x, y, z) = \sin(x + 2y) + \sqrt{xyz}, \quad (x_0, y_0, z_0) = \left(\frac{\pi}{2}, \frac{3\pi}{2}, 3\right), \quad \mathbf{u} = \left(\frac{4}{5}, \frac{3}{5}, 0\right).$$

Task 5: Using differentials calculate approximate values of the expressions:

$$a) (1,02)^3 \cdot (0,997)^2; \quad b) \sqrt[3]{(2,93)^3 + (4,05)^3 + (4,99)^3}; \quad c) (2,97)^2 \cdot e^{0,05}; \quad d) (1,01)^{2,98}; \quad e) \frac{\cos 0,05}{1,96}.$$

Task 6: Calculate all the second-order partial derivatives:

$$a) f(x, y) = xy + \frac{x^2}{y^3}; \quad b) f(x, y) = \sin(x^2 + y^2); \quad c) f(x, y) = xe^{xy};$$

$$d) f(x, y, z) = e^{3x+4y} \cos 5z; \quad e) f(x, y, z) = \ln(x^2 + y^4 + z^6 + 1); \quad f) f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}.$$