LIST 2. Theory of Sets. Mathematical Induction.

1. Draw these sets on x-axis.

$A \cap B,\quad A \cup B,\quad A \setminus B,\quad B \setminus A,\quad A \div B,\quad A^C,\quad B^C,\quad A^C \cap B^C,\quad A^C \cup B^C$

Also, draw the Cartesian products $A \times B,\quad B \times A$ on xy-plane.

$(a)\;\; A = \{x \in R : x^2 + x - 20 > 0\}\;\; and \;\; B = \{x \in R : x^2 \le 1\};$

$(b)\;\; A = (-1,2)\;\; and \;\; B = \langle 1,4\rangle;$

$(c)\;\; A = \{x \in N : x^2 - 3x - 4 \le 0\}\;\; and \;\; B = \{x \in R : x^2 - 4x + 3 \ge 0\};$

$(d)\;\; A = \{x \in R : x^2 - 4 < 0\}\;\; and \;\; B = \{-3,0,1,2,4\}.$

2. For the given A_n, n-natural number, find:

$$\bigcup_{n=1}^{\infty} A_n \text{ and } \bigcap_{n=1}^{\infty} A_n$$

$(a)\;\; A_n = \left\{x \in R : -2 + \frac{1}{n} \le x < 3 - \frac{1}{n}\right\},\qquad$ (b) $A_n = \left\{x \in R : \frac{1}{n} \le x \le 3 + \frac{1}{n}\right\},$

$(c)\;\; A_n = \left\{x \in R : (-1)^n \le x \le 1 + \frac{4}{n}\right\},\qquad (d)\;\; \{x \in R : -2n \le x \le 2n\},\qquad (e)\;\; A_n = \left(-\frac{1}{n}; \frac{1}{n}\right).$

3. Using the mathematical induction prove the formulas:

$(a)\;\; (1+a)^n \ge 1 + na + \frac{n(n-1)}{2}a^2 \qquad n \in N \cup \{0\}, \qquad a \in R^+ \cup \{0\};$

$(b)\;\; (1+a)^n \ge 1 + na \qquad n \ge 1, \qquad a > -1;$

$(c)\;\; \sum_{k=1}^{n} \sin kx = \frac{1}{\sin\frac{x}{2}} \sin\frac{(n+1)x}{2} \sin\frac{nx}{2} \qquad x \in R, \qquad x \ne m\pi, \qquad m \in Z, \qquad n \in N$

<u>Remark:</u> Solve the example (c) using the formulas:

$\sin 2\alpha = 2\sin\alpha\cos\alpha$ and $\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$.