LIST 2. Triple Integrals.

1. Find the triple integral: $\iiint\limits_Q \frac{x\,dxdydz}{yz}$, *where* $Q=[1,2]\times[1,e]\times[1,e]$.

2. The triple integral change to the sum of multiplications of Riemann integrals:

$$\iiint\limits_Q z\ln\left(x^y y^x\right)dxdydz, \quad where \quad Q=[1,e]\times[1,e]\times[0,1].$$

3. The triple integral $\iiint\limits_V f(x,y,z)dxdydz$ change to the iterated integral, where V is area limited by the surfaces:

a) $x^2+y^2+z^2=25$, $z=4$ $(z\geq 4)$; b) $z=x^2+y^2$, $z=\sqrt{20-x^2-y^2}$.

4. In the integrals below change the order of integration:

a) $\int\limits_{-2}^{2}dx\int\limits_{-\sqrt{4-x^2}}^{0}dy\int\limits_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}}f(x,y,z)dz$; b) $\int\limits_0^3 dz\int\limits_{-\sqrt{z}}^{\sqrt{z}}dx\int\limits_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}}f(x,y,z)dy$.

5. Calculate the triple integrals:

a) $\iiint\limits_V \sqrt{x^2+y^2}\,dxdydz$, *where* $V:\ x^2+y^2=z^2,\ z=1$;

b) $\iiint\limits_V y\cos(x+z)dxdydz$, *where* $V:\ y=\sqrt{x},\ y=0,\ z=0,\ x+z=\frac{\pi}{2}$;

c) $\iiint\limits_V \left(x^2+y^2\right)dxdydz$, *where* $V:\ r^2\leq x^2+y^2+z^2\leq R^2,\ z\geq 0$;

d) $\iiint\limits_V \sqrt{x^2+y^2+z^2}\,dxdydz$, *where* $V:\ x^2+y^2+z^2\leq x$;

e) $\iiint\limits_V \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}}\,dxdydz$, *where* $V:\ \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\leq 1$.

6. Find the volume of the solids limited by the surfaces:

a) $x^2+z^2=a^2$, $x+y=-a$, $x+y=a$, $x-y=-a$, $x-y=a$; b) $az=a^2-x^2-y^2$, $z=a-x-y$, $x=0$, $y=0$, $z=0$ $(a>0)$;

c) $x^2+y^2+z^2=a^2$, $z^2=x^2+y^2$ $(z^2\leq x^2+y^2)$; d) $x^2+y^2+z^2=4$, $x^2+y^2=3z$.

7. Find the coordinates of the center of mass of the homogenous area (density $\sigma(x,y,z)=1$):

a) $V=\left\{(x,y,z)\in R^3:\ \frac{x^2}{a^2}+\frac{y^2}{b^2}\leq\frac{z^2}{c^2},\ z\leq c\right\}$;

b) $V=\left\{(x,y,z)\in R^3:\ z\leq x^2+y^2,\ x+y\leq a,\ x\geq 0,\ y\geq 0,\ z\geq 0\right\}$;

c) $V=\left\{(x,y,z)\in R^3:\ x^2+z^2\leq a^2,\ y^2+z^2\leq a^2,\ (z>0)\right\}$

8. Calculate the moment of inertia of the solids limited by the surfaces:

a) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1,\ x=0,\ y=0,\ z=0$ with respect to xy-plane;

b) $\frac{x^2}{4}+\frac{y^2}{9}=\frac{2}{5}z,\ \frac{x}{2}+\frac{y}{3}=\frac{z}{5}$ with respect to xz-plane;

c) $z^2=2ax,\ x^2+y^2=ax,\ z=0$ with respect to z-axis.

In each case density $\sigma(x,y,z)=1$.