LIST 3. Algebraic structures.

1. For the algebraic system $(A,\cdot)$, where A={-1,1}, and $\cdot$ is simply multiplication, prove that it is a commutative group. Build a table of operations.

2. For the algebraic system $(Z_4,\oplus)$, where Z_4={0,1,2,3}, and $\oplus$ is the addition modulo 4, build a table of operations. Prove that it is a commutative group.

3. For the algebraic system $(Z_5,\oplus,\otimes)$, where Z_5={0,1,2,3,4}, $\oplus$ and $\otimes$ are respectively the addition and multiplication modulo 5, build a table of operations. Prove that it is a commutative ring with unit.

4. Explain why the algebraic system $(Z_5,\oplus,\otimes)$, where Z_5={0,1,2,3,4}, $\oplus$ and $\otimes$ are respectively the addition and multiplication modulo 5, is the algebraic field but the algebraic structure $(Z_4,\oplus,\otimes)$, where Z_4={0,1,2,3}, $\oplus$ and $\otimes$ are respectively the addition and multiplication modulo 4, is not the algebraic field.

5. Prove that set K={x$\in$R: $x=a+b\sqrt{2}$ a,b$\in$Q} with operations + and $\cdot$ is the algebraic field.