LIST 3. Linear Differential Equations with Constant Coefficients n^{th} Order.

Task 1: Find the general solutions of the homogenous linear equations:

$\quad$ a) $y''-6y'+8y=0$; $\qquad$ b) $y''-y'+y=0$; $\qquad$ c) $y''+4y'+4y=0$;
$\qquad$ d) $y''+3y'+2y=0$; $\qquad\qquad$ e) $y''-2y'=0$; $\qquad$ f) $y''+4y=0$;
g) $y^{(4)}+2y'''+2y''+2y'+y=0$; $\quad$ h) $y'''+y''+y'+y=0$; $\quad$ i) $y^{(7)}+6y^{(6)}+12y^{(5)}+8y^{(4)}=0$.

Task 2: Find the general solutions of the non-homogenous linear equations using method of undetermined coefficients:

$\quad$ a) $y''-y=x^2-x+1$; $\qquad$ b) $y''+y'=3$; $\qquad$ c) $y''-2y'-3y=-4e^x+3$;
$\qquad$ d) $y''+y=6\sin2x$; $\qquad$ e) $y''-3y'=e^{3x}-18x$; $\qquad$ f) $y''+4y=\sin2x$;
$\qquad$ g) $y^{(4)}-7y'''+12y''=x$; $\qquad$ h) $y'''+2y''+y'=8e^x$; $\qquad$ i) $y^{(4)}-y''=2\sin x$.

Task 3: Find the general solutions of the non-homogenous linear equations using method of variation of parameters:

$\quad$ a) $y''-4y=\dfrac{e^{2x}}{e^{2x}+1}$; $\qquad$ b) $y''+y=\dfrac{2}{\sin^3 x}$; $\qquad$ c) $y'''+y=\dfrac{x^3-6}{x^4}$.

Task 4: Find the particular solutions of the homogenous or non-homogenous linear equations satisfying given IVP:

a) $y^{(4)}-y=0$; $\qquad$ $x_0=0$, $y_0=1$, $y_1=1$, $y_2=1$, $y_3=1$;
b) $y''+4y'+4y=3e^{-2x}$; $\qquad$ $(0;0;0)$;
c) $y'''-y'=0$; $\qquad$ $y(2)=0$, $y'(2)=0$, $y''(2)=0$;
d) $y''+y=x$; $\quad$ $y(0)=1$, $\quad$ $y'(\dfrac{\pi}{2})=\dfrac{\pi}{2}$.