

LIST 3. Scalar Line Integral.

1. Calculate scalar line integrals on lines K:

a) $\int_K (x^2 + y) dl$, K: the interval from point A(0,2) to point B(2,4);

b) $\int_K xy dl$, K: the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ and $x \geq 0, y \geq 0$.

2. Calculate scalar line integrals on lines K given in polar coordinates:

a) $\int_K \frac{dl}{(x^2 + y^2)^{\frac{3}{2}}}$, K: the hyperbolic spiral $r = \frac{1}{\varphi}, \sqrt{3} \leq \varphi \leq 2\sqrt{2}$;

b) $\int_K \sqrt{2y^2 + z^2} dl$, K: the logarithmic spiral $r = ae^{\varphi}$ from the point A(a,0) to the point B($ae^{2\pi}$,0).

3. Calculate scalar line integrals on lines K:

a) $\int_K (x+z) dl$, K: $x=t, y=\frac{3t^2}{\sqrt{2}}, z=t^3, 0 \leq t \leq 1$; b) $\int_K \sqrt{2y^2 + z^2} dl$, K: the circle: $x^2 + y^2 + z^2 = a^2, x=y$.

4. Calculate the center of mass of the homogenous cycloid:

$$x = a(t - \sin t) \quad y = a(1 - \cos t) \quad 0 \leq t \leq \pi$$

5. Calculate the center of mass and the moment of inertia with respect to the point O(0;0;0) of the homogenous helical curve:

$$x = e^t \cos t \quad y = e^t \sin t \quad z = e^t \quad 0 \leq t \leq \pi/2$$

6. Calculate the center of mass of homogenous spherical triangle:

$$x^2 + y^2 + z^2 = a^2, \quad x > 0, \quad y > 0, \quad z > 0.$$