LIST 4. Complex Numbers

Task 1. Calculate:

(a) $(4+3i)-(2-4i)$, (b) $(2-3i)(3+4i)$, (c) $(4+i)(1-2i)(-3+2i)$,

(d) $(\sqrt{5}-i\sqrt{3})\cdot(\sqrt{5}+i\sqrt{3})$, (e) $\dfrac{3+i}{2-i}$, (f) $\dfrac{(4+5i)(2-i)}{3+i}$, (g) $\dfrac{i\cdot(1-i\sqrt{3})^2}{\sqrt{3}+i}$.

Task 2. For numbers $z_1 = 2-3i$ and $z_2 = -1+2i$ calculate

(a) $z_1+\bar{z}_2$, (b) $z_1\cdot\bar{z}_2$, (c) $z_1^2-z_2^2$.

Task 3. Find real numbers x and y satisfying the given equations:

(a) $(3-2i)\cdot x+(2+i)\cdot y=4-5i$; (b) $\dfrac{1+yi}{x-2i}=3i-1$; (c) $\dfrac{x+yi}{x-yi}=\dfrac{9-2i}{9+2i}$.

Task 4. In the set of complex numbers solve the given equations: .

(a) $z^2-4z+13=0$; (b) $2z+\bar{z}=6-5i$; (c) $(z+2)^2=(\bar{z}+2)^2$;

(d) $\dfrac{2+i}{z-1+4i}=\dfrac{1-i}{2z+i}$; (e) $z\cdot\bar{z}+(z-\bar{z})=10-6i$.

Task 5. Find the polar and exponential forms of complex numbers:

(a) -5, (b) $-2i$, (c) $-1+i\sqrt{3}$, (d) $-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$, (e) $\dfrac{2-2i}{i}$, (f) $-\sqrt{3}-i\sqrt{3}$.

Task 6. Using the geometric interpretation of modulus of a complex number draw the set of numbers z satisfying the given conditions:

(a) $|z|\le 2$, (b) $1<|z|<3$, (c) $\dfrac{\pi}{6}\le\arg z\le\dfrac{\pi}{3}$, (d) $0\le \mathrm{Re}\,z\le 2$, (e) $\mathrm{Im}\,z=-2$, (f) $|z-2|\le 2$,

(g) $|z+3i|=1$, (h) $|z+2-i|\ge 3$, (i) $\dfrac{4}{z}=\bar{z}$, (j) $\left|\dfrac{z-2i}{z+1}\right|=1$, (k) $2\le|iz-5|<3$.

Task 7. Calculate:

(a) i^{15}, (b) $(1+i)^5$, (c) $(-1+i)^7$, (d) $\left(\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i\right)^9$, (e) $\left(1+\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)^4$, (f) $\dfrac{(1-i)^2e^{\pi i}}{(1+i)^3e^{(\pi/4)i}}$,

(g) $e^{1+(\pi/2)i}$, (h) $(-1+i\sqrt{3})^5$, (i) $\sqrt[4]{1}$, (j) $\sqrt[3]{1}$, (k) $\sqrt[3]{-i}$, (l) $\sqrt[4]{i}$, (m) $\sqrt[3]{-1}$, (n) $\sqrt[4]{16}$, (o) $\sqrt{1-\sqrt{3}i}$.

Task 8. Using the de Moivre's law find formulas for $\sin3\alpha$ and $\cos3\alpha$ as a functions of $\sin\alpha$ i $\cos\alpha$.

Task 9. Using the definition calculate the square roots of the given complex numbers:

(a) z=3+2i; (b) z=-9-2i ; (c) z=7+4i.

Task 10. In the set of complex numbers solve the polynomial equations:

(a) $z^2+4z+8=0$, (b) $z^3-z^2+z-1=0$, (c) $z^4+3z^2-4=0$, (d) $z^3-4z^2+6z-4=0$,

(e) $z^2+32=0$, (f) $z^8+15z^4-16=0$, (g) $z^4+1=0$, (h) $z^2-4z+13=0$,

(i) $z^2-3z+3+i=0$, (j) $z^4-2z^2+4=0$, (k) $z^2+(1+4i)z-3-i=0$,

(l) $z^4+(15+7i)z^2+8-15i=0$, (m) $z^2+2(1+i)z+2i=0$.

Task 11. Using the Euler's formula: $e^{ix}=\cos x+i\sin x$ find expressions for *sinx* and *cosx* as exponential functions with complex exponents.