LIST 4. Definite Integrals.

Task 1: Calculate definite integrals:

a. $\int_{-1}^{2} x(1+x^3)dx$;

b. $\int_{1}^{2} \left(\frac{1}{x^3} - \frac{1}{x^2} + x^{-4} \right)dx$;

c. $\int_{1}^{e} x\ln x dx$;

d. $\int_{0}^{\frac{\pi}{2}} e^{2x} \cos x dx$;

e. $\int_{0}^{1} x\sqrt{1+x}dx$;

f. $\int_{0}^{\frac{1}{2}\ln 3} \frac{e^x}{1+e^{2x}} dx$.

Task 2: Calculate the average values of functions given below in the specified intervals:

a. $f(x) = x\sin x$, $[0; \pi]$;

b. $f(x) = x\sqrt{1-x^2}$, $\left[0; \frac{1}{2} \right]$.

Task 3: Calculate areas limited by curves:

a. y^2=-x, y=x-6, y=-1, y=4;

b. $y = x^2 - 4$, $y = -x^2 + 9$;

c. $y = \frac{1}{x^2}$, $y = x$, $y = 4$;

d. $y = x^3$, $y = 4x$ (y>0);

e. $x^2 + y^2 = 4$, $y = 2x - x^2$, $x = 0$ (x,y≥0);

f. $y = 4 - x^2$, $y = x^2 - 2x$.

g. $y = x^2 + 2x$, $y = x + 2$;

h. $y = \frac{27}{x^2 + 9}$, $y = \frac{x^2}{6}$;

i. $y = \ln(x+1)$, $y = \ln x$, $x = 2$, $y = 0$;

j. $y = \ln x$, $y = \ln(3-x)$, $y = 0$.

k. $y = e^x$, $y = e^{2x} - 2$, $x = 0$;

l. $y = e^x$, $y = e^{-x}$, $y = 2$.

Task 4: Calculate the length of the given curves:

a. $y = 2\sqrt{x^3}$, $0 \le x \le \sqrt{10}$;

b. $y = e^x$, $\frac{1}{2}\ln 2 \le x \le \frac{1}{2}\ln 3$;

c. $y = 1 - \ln\cos x$, $0 \le x \le \frac{\pi}{4}$.

Task 5: Calculate the volume of solids of revolution (rotation around the x-axis):

a. $1 \le x \le 4$, $\frac{4}{x} \le y \le 5 - x$;

b. $0 \le x \le \frac{\pi}{2}$, $0 \le y \le \sin x + \cos x$.

Task 6: Calculate lateral area of solids of revolution (rotation around the x-axis):

a. $f(x) = \sqrt{4-x^2}$, $-1 \le x \le 1$;

b. $f(x) = \sqrt{x}\left(1 - \frac{x}{3} \right)$, $1 \le x \le 3$.