LIST 4. Differential Equations.

Task 1: Solve the equations by separating the variables:

a) $y' = e^{x-y}$;

b) $2x\sqrt{1-y^2}\,dx + ydy = 0$;

c) $y' = \dfrac{y-1}{x+1}$;

d) $y' = \dfrac{\sqrt{y}}{\sqrt{x}}$;

e) $y' = -y\sin x$;

f) $\left(y^2 + xy^2\right)dx + \left(x^2 - yx^2\right)dy = 0$;

g) $\left(1+y^2\right)\left(e^{2x}dx - e^y dy\right) - \left(1+y\right)dy = 0$;

h) $\dfrac{dx}{\sqrt{1-x^2}} + \dfrac{dy}{\sqrt{1-y^2}} = 0$.

Task 2: Find solutions of equations with separated variables satisfying the initial value problems:

a) $dx - \sqrt{1-x^2}\,dy = 0$, $(x_0; y_0)=(1; \dfrac{\pi}{2})$;

b) $\left(1-x\right)dy - ydx = 0$, $(x_0; y_0)=(0;1)$;

c) $x\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$, $(x_0; y_0)=(0;0)$;

d) $xdy - ydx = 0$, $(x_0; y_0)=(1;1)$.

Task 3: Solve the first order linear differential equations:

a) $y' - y\sin x = \sin x\cos x$;

b) $\left(1+x^2\right)y' - 2xy = \left(1+x^2\right)^2$;

c) $y' - \dfrac{1}{x}y = x$;

d) $y' + 2xy = 2xe^{-x^2}$;

e) $y' + y = 2e^x$.

Task 4: Find solutions of the first order linear differential equations satisfying the initial value problems $y(x_0)=y_0$:

a) $y' + \dfrac{3}{x}y = \dfrac{2}{x^3}$, $x_0=1$, $y_0=1$;

b) $xy' = x - y$, $x_0=1$, $y_0=3$;

c) $y' - 2xy = x$, $x_0=1$, $y_0=1$;

d) $xy' = x + 2y$, $x_0=1$, $y_0=2$.

Task 5: Find the general solutions of the homogeneous linear equations:

a) y''-6y'+8y=0;
b) y''-y'+y=0;
c) y''+4y'+4y=0;
d) y''+3y'+2y=0;
e) y''-2y'=0;
f) y''+4y=0;
g) $y^{(4)}+2y'''+2y''+2y'+y=0$;
h) y'''+y''+y'+y=0;
i) $y^{(7)}+6y^{(6)}+12y^{(5)}+8y^{(4)}=0$.

Task 6: Find the general solutions of the non-homogeneous linear equations using method of undetermined coefficients or method of variation of parameters:

a) $y''-y=x^2-x+1$;
b) y''+y'=3;
c) $y''-2y'-3y=-4e^x+3$;
d) y''+y=6sin2x;
e) $y''-3y'=e^{3x}-18x$;
f) y''+4y=sin2x;
g) $y^{(4)}-7y'''+12y''=x$;
h) $y'''+2y''+y'=8e^x$;
i) $y^{(4)}-y''=2\sin x$.