LIST 4. Vector Line Integral.

1. Calculate vector line integrals on curves:

a) $\int_K (y-z)dx+(z-x)dy+(x-y)dz$, K: $\begin{cases} x = a\cos t \\ y = a\sin t \quad 0 \le t \le 2\pi \\ z = bt \end{cases}$;

b) $\int_K ydx+zdy+xdz$, K: $\begin{matrix} x = R\cos\alpha\cos t \\ y = R\cos\alpha\sin t \quad 0 \le t \le 2\pi \quad R,\alpha - const. \\ z = R\sin\alpha \end{matrix}$;

c) $\int_{\widehat{OA}} xydx+yzdy+xzdz$, OA: $x^2+y^2+z^2=2Rx$ $z=x$ $(y\ge 0)$.

2. Find the potential of vector field W=[P,Q,R] and the work done by W in moving an object between two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$:

a) P=0, Q=0, R=-mg (force of gravity);

b) $P=-\frac{mx}{r^3}$, $Q=-\frac{my}{r^3}$, $R=-\frac{mz}{r^3}$, $r=\sqrt{x^2+y^2+z^2}$, m-mass (Newtonian force of attraction);

c) $P=-k^2x$, $Q=-k^2y$, $R=-k^2z$, k-const., the starting point A is on the sphere $x^2+y^2+z^2=R^2$ and the end point on the sphere $x^2+y^2+z^2=r^2$ (R>r).

3. Check if the given integrals are independent on path of integration and then calculate them:

a) $\int_{(1,0,-3)}^{(6,4,8)} xdx+ydy-zdz$; b) $\int_{(1,1,1)}^{(a,b,c)} yzdx+zxdy+xydz$; c) $\int_{(0,0,0)}^{(3,4,5)} \frac{xdx+ydy+zdz}{\sqrt{x^2+y^2+z^2}}$

4. Calculate the vector line integrals:

a) $\int_K (x^2-2xy)dx+(2xy+y^2)dy$, K: $y=x^2$ the starting point A(1,1), the end point B(2,4);

b) $\int_K (2a-y)dx+xdy$, K: the first arc of cycloid: $\begin{matrix} x = a(t-\sin t) \\ y = a(1-\cos t) \end{matrix}$.

5. Using the Green's theorem calculate the integrals on the closed curves:

a) $\oint_K \sqrt{x^2+y^2}\,dx + y\left[xy+\ln\left(x+\sqrt{x^2+y^2}\right)\right]dy$, K: the bank of any figure S;

b) $\oint_K 2\left(x^2+y^2\right)dx+(x+y)^2dy$, K: the broken line ABCA; A(1,1), B(2,2), C(1,3);

c) $\oint_K -x^2ydx+xy^2dy$, K: the circle $x^2+y^2=R^2$.

6. Using the Green's theorem calculate surface area of the figures limited by the curves:

a) ellipse: x=acost, y=bsint;

b) asteroid: $x=a\cos^3 t$, $y=b\sin^3 t$;

c) cardioid: x=a(2cost-cos2t), y=b(2sint-sin2t).