LIST 5. Scalar Surface Integrals.

Task. 1. Calculate scalar surface integrals:

a) $\iint\limits_S (x^2 + y^2)dS;$ where $S: \sqrt{x^2 + y^2} \leq z \leq 1$;

b) $\iint\limits_S (8 - 2z)dS;$ where $S: z = 4 - \dfrac{1}{2}x^2 - \dfrac{1}{2}y^2$ for $z > 0$;

c) $\iint\limits_S x^2 y^2 dS;$ where $S: z = \sqrt{R^2 - x^2 - y^2}$;

d) $\iint\limits_S \dfrac{dS}{r};$ where $S: z = xy$ for $x^2 + y^2 \leq R^2$, r – distance between S and z-axis;

e) $\iint\limits_S (2x + 1)dS;$ where $S: x = \sqrt{4 - y^2}$ for $0 \leq z \leq 1$.

Task. 2. Calculate the mass of this part of the paraboloid $z = \dfrac{1}{2}(x^2 + y^2)$, which is limited by the planes z = 0 and z = 1 and the density at each point is equal to the third coordinate of point of the surface S.

Task. 3. Calculate the mass of this part of the plane x+y+z=1, which is limited by cartesian coordinates planes (x≥0, y≥0, z≥0) with density $\rho(x, y, z) = \dfrac{1}{(1 + x + y)^2}$.

Task. 4. Calculate the mass of the paraboloid $z = x^2 + y^2$ located below the plane z=1 with density $\rho(x, y, z) = |xyz|$.

Task. 5. Calculate the mass of the surface: $\begin{array}{l} x = u\cos v \\ y = u\sin v \\ z = v \end{array}$ $\begin{array}{l} 0 \leq u \leq a \\ 0 \leq v \leq 2\pi \end{array}$, if $\rho(x, y, z) = z$.

Task. 6. For the this part of homogenous surface $z = \sqrt{x^2 + y^2}$ which is cut out by cylinder $x^2 + y^2 = 2x$ calculate moment of inertia with respect to z-axis.

Task. 7. For the this part of homogenous surface $z = x + y$ which is cut out by cylinder $x^2 + y^2 = 4$ calculate the static moment with respect to the xy-plane.

Task. 8. Calculate the center of mass of the homogenous sphere $x^2 + y^2 + z^2 = a^2$ limited by the planes z=h and z=a, 0<h<a.