LIST 6. Matrices

1. Calculate:

a) $2\begin{bmatrix} 0 & 4 \\ 5 & -1 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}$;

b) $\begin{bmatrix} 0 & 4 \\ 5 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 5 & 3 \\ 2 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & -2 \\ 3 & -1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & -2 \\ 3 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 & 3 \\ 2 & -3 & 1 \end{bmatrix}^T$;

d) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$.

2. Solve given matrix equations and systems of matrix equations:

a) $X + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \frac{1}{2}\left(X - \begin{bmatrix} 0 & 0 & 2 \\ 0 & 4 & 0 \end{bmatrix} \right)$;

b) $\begin{cases} X + Y = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \\ X - Y = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} \end{cases}$.

3. Calculate a few first powers of the matrix A (i.e. A^2, A^3, ...):

a) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$;

b) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

4. Building adequate systems of equations, find all matrices X that satisfying complex matrix equation

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}^T \cdot X = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$; b) $X - iX^T = \begin{bmatrix} 4i & 0 \\ 6 - 2i & -2 \end{bmatrix}$; c) $X^2 = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$;

d) $X \cdot X^T = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$, where X is 2×2 dimension matrix.