LIST 6. Vector Surface Integrals.

Task. 1. Calculate vector surface integrals:

a) $\iint\limits_S xdydz - dxdz + ydxdy$; S: part of the outer side of the pyramid O(0,0,0), A(1,0,0), B(0,2,0), C(1,0,0) without the side AOC;

b) $\iint\limits_S xdydz + ydxdz + zdxdy$; S: the inner side of the sphere $x^2 + y^2 + z^2 = R^2$.

c) $\iint\limits_S dydz - 2dxdz + x^3dxdy$; S: the outer side of the surface consisting of $z = \sqrt{x^2 + y^2}$, $x^2 + y^2 = 4z - 4$, $x=0$ and $y=0$ $(x\geq 0,\ y\geq 0,\ z\geq 0)$;

d) $\iint\limits_S xzdydz + x^2ydxdz + y^2zdxdy$; S: the outer side of the surface consisting of the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 1$ $(x\geq 0,\ y\geq 0,\ z\geq 0)$;

Task. 2. Check Gauss-Ostrogradzky theorem for:

a) $\iint\limits_S xzdydz + xydxdz + yzdxdy$; S: the outer side of the surface limited by cylinder $x^2 + y^2 = R^2$ and planes x=0, y=0, z=0, z=k (x≥0, y≥0);

b) $\iint\limits_S (x - y + z)dydz + (y - z + x)dxdz + (z - x + y)dxdy$; S: the outer side of the surface $|x - y + z| + |y - z + x| + |z - x + y| = 1$

Task. 3. Using Stokes' theorem calculate integrals:

a) $\int\limits_K x^2y^3dx + dy + zdz$, K: the positively oriented circle $x^2+y^2=R^2$ $z=0$;

b) $\int\limits_K xdx + (x + y)dy + (x + y + z)dz$, K: $\begin{cases} x = a\sin t \\ y = a\cos t \\ z = a(\sin t + \cos t) \end{cases}$ $0 \leq t \leq 2\pi$.