LIST 7. Determinants

1. Calculate determinants:

a) $\begin{vmatrix} -3 & 2 \\ 8 & -5 \end{vmatrix}$; b) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix}$; c) $\begin{vmatrix} 1 & i & 1+i \\ -i & 1 & 0 \\ 1-i & 0 & 1 \end{vmatrix}$.

2. Using the Laplace's expansion calculate the mentioned below determinants:

a) $\begin{vmatrix} i & 1+i & 2 \\ 1-2i & 3 & -i \\ -4 & 1-i & 3+i \end{vmatrix}$, along the 3rd column; b) $\begin{vmatrix} -1 & 2 & -3 & 4 \\ 0 & 5 & 3 & -7 \\ 1 & 3 & -5 & 9 \\ 2 & -2 & 4 & 6 \end{vmatrix}$, along the 2nd row.

3. Using the Laplace's expansion calculate the mentioned below determinants. Select the column or row with the largest number of zeros:

a) $\begin{vmatrix} 3 & -2 & 0 & 5 \\ -2 & 1 & -2 & 2 \\ 0 & -2 & 5 & 0 \\ 5 & 0 & 3 & 4 \end{vmatrix}$; b) $\begin{vmatrix} 3 & 2 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 & 0 \\ 0 & 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 3 & 2 \\ 2 & 0 & 0 & 0 & 3 \end{vmatrix}$.

4. Using elementary operations on rows or columns, calculate determinants:

a) $\begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 5 \\ -4 & 0 & 6 \end{vmatrix}$; b) $\begin{vmatrix} 4 & 2 & 1 & 1 \\ 1 & -1 & 0 & 2 \\ 3 & 0 & 1 & 3 \\ 2 & 2 & 0 & 3 \end{vmatrix}$; c) $\begin{vmatrix} 1 & 2 & -1 & 0 & 3 \\ 2 & 4 & 5 & 1 & -6 \\ -1 & -2 & 3 & 0 & -2 \\ -2 & -2 & 1 & -1 & 1 \\ 2 & 4 & -2 & 0 & 3 \end{vmatrix}$.

5. Using formula describing the inverse matrix calculate the inverse matrix A^{-1}:

a) $A = \begin{bmatrix} 3 & -5 \\ 6 & 2 \end{bmatrix}$; b) $A = \begin{bmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{bmatrix}$.

6. Applying Gaussian Elimination Algorithm calculate the inverse matrix A^{-1}:

a) $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$; b) $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix}$.

7. Solve given matrix equations calculating the inverse matrices:

a) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \cdot X \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$; b) $\left(\begin{bmatrix} 0 & 3 \\ 5 & -2 \end{bmatrix} + 4 \cdot X \right)^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$;

c) $3 \cdot X + \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \cdot X$.