Physical interpretation of triple integrals

Notation: ρ(x,y,z) – mass distribution on space area V.

$$M = \iiint_V \rho(x,y,z)\,dxdydz \qquad mass \quad of \quad solid \quad V$$

The static moments

The moments of inertia

a. with respect to point (0,0,0)

$$M_{(0,0,0)} = \iiint_V \sqrt{x^2+y^2+z^2}\,\rho(x,y,z)\,dxdydz$$

$$B_{(0,0,0)} = \iiint_V \left(x^2+y^2+z^2\right)\rho(x,y,z)\,dxdydz$$

b. with respect to the axis of the coordinate system

$$M_{OX} = \iiint_V \sqrt{y^2+z^2}\,\rho(x,y,z)\,dxdydz$$

$$M_{OY} = \iiint_V \sqrt{x^2+z^2}\,\rho(x,y,z)\,dxdydz$$

$$M_{OZ} = \iiint_V \sqrt{x^2+y^2}\,\rho(x,y,z)\,dxdydz$$

$$B_{OX} = \iiint_V \left(y^2+z^2\right)\rho(x,y,z)\,dxdydz$$

$$B_{OY} = \iiint_V \left(x^2+z^2\right)\rho(x,y,z)\,dxdydz$$

$$B_{OZ} = \iiint_V \left(x^2+y^2\right)\rho(x,y,z)\,dxdydz$$

c. with respect to the planes of the coordinate system

$$M_{OXY} = \iiint_V z\rho(x,y,z)\,dxdydz$$

$$M_{OYZ} = \iiint_V x\rho(x,y,z)\,dxdydz$$

$$M_{OXZ} = \iiint_V y\rho(x,y,z)\,dxdydz$$

$$B_{OXY} = \iiint_V z^2\rho(x,y,z)\,dxdydz$$

$$B_{OXZ} = \iiint_V y^2\rho(x,y,z)\,dxdydz$$

$$B_{OYZ} = \iiint_V x^2\rho(x,y,z)\,dxdydz$$

The center of mass $P(x_0;y_0;z_0)$:

$$x_0 = \frac{M_{OYZ}}{M} \qquad y_0 = \frac{M_{OXZ}}{M} \qquad z_0 = \frac{M_{OXY}}{M}$$